Z-Scores

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• Z-scores are a way of standardising a score with respect to the other scores in the group.
• This is done by taking account of the mean and SD of the group.
• A Z-score expresses a particular score in terms of how many Standard Deviations it is away from the mean.

By converting a raw score to a z-score, we are expressing that score on a z-score scale, which always has a mean of 0 and a standard deviation of 1.

In short, we are re-defining each raw score in terms of how far away it is from the group mean.

Calculating a Z-score

• First, we find the difference between the raw score and the mean score (this tells us how far away the raw score is from the average score).
• Second, we divide by the standard deviation (this tells us how many standard deviations the raw score is away from the average score).

\[ Z = \frac{X - \bar{X}}{S} \]

\[ \text{Mean} = 100, \text{SD} = 20 \]

\[ \text{Mean} = 60, \text{SD} = 5 \]

\[ \text{Mean} = 0, \text{SD} = 1 \]
Advantages of Using Z-scores

- **Clarity**: The relationship between a raw score and the distribution of scores is much clearer. It is possible to get an idea of how good or bad a score is relative to the entire group.

- **Comparison**: You can compare scores measured on different scales.

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Area Under The Curve: We know various properties of the normal distribution.

- By converting to a normal distribution of z-scores, we can see how many scores should fall between certain limits.
- We can, therefore, calculate the probability of a given score occurring.

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Area Under the Normal Curve

- In most Statistics text books you can find a table of numbers labelled *area under the normal curve*.
- This table allows us to discover things about any set of scores provided that we first convert them to z-scores.

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- **Area between the mean and z**: This part of the table tells us the proportion of scores that lie between the mean and a given z-score (this proportion is the area under the curve between those points).

- **Area beyond z**: This part of the table tells us the proportion of scores that were greater than a given z-score.

These areas can be used to find out:

- The proportion of Scores that were greater than a particular score on a test.
- What proportion of scores lie between the mean and a given test score.
- What proportion lie between two scores.
Example 1

- A social-skills scale had a mean of 100 and a standard deviation of 15.
- 263 people at RH were tested.
- A psychology Student scores 130.
- What proportion of people got a higher score than this? How many people is this?

Convert the Raw Score to a Z-score

\[ z = \frac{X - \bar{X}}{s} \]

\[ z = \frac{130 - 100}{15} \]

\[ z = \frac{30}{15} \]

\[ z = 2 \]

Look up the proportion in the z-score table

- The diagram shows that we are interested in the area above 130 (shaded).
- Look in column labelled area above \( z \).
- When \( z = 2 \), area beyond = 0.0228.

Percentage = \( 100 \times 0.0228 = 2.28\% \)

Conclusion

- 2.28% of people had better social skills than our psychology student.
- We can work out how many people this was by multiplying the proportion by the number of scores collected:

\[ 263 \times 0.0228 = 6 \text{ people} \]

Example 2

- A social-skills scale had a mean of 100 and a standard deviation of 15.
- 263 people at RH were tested.
- A statistic lecturer scores of 60.
- What proportion of people got a lower score than this? How many people is this?
Convert the Raw Score to a Z-score

\[ z = \frac{X - \bar{X}}{s} \]

1. \[ z = \frac{60 - 100}{15} \]
2. \[ z = \frac{-40}{15} \]
3. \[ z = -2.67 \]

Look up the proportion in the z-score table

- The diagram shows that we are interested in the area below 40 (shaded).
- Look in column labelled area above \( z \).
- when \( z = 2.67 \), area beyond \( = 0.0038 \).

\[ \text{Percentage} = 100 \times 0.0038 = 0.38\% \]

Conclusion

- 0.38\% of people had worse social skills than our statistic lecturer.
- We can work out how many people this is by multiplying the proportion by the number of scores collected:

\[ 263 \times 0.0038 = 1 \text{ person} \]

Example 3

- 130 Students' degree percentages were recorded.
- The mean percentage was 58\% with a standard deviation of 7.
- What proportion of students received a 2:1? How many people is this?
- Hint 2:1 = between 60\% and 69\%
Convert the Raw Scores to Z-scores

\[ z_{60} = \frac{60 - 58}{7} = 0.29 \]

\[ z_{69} = \frac{69 - 58}{7} = 1.57 \]

Look up the proportions in the z-score table

- The diagram shows that we are interested in the area above both scores.
- Look in column labelled area above z.
- when \( z = 0.29 \), area beyond = 0.386.
- when \( z = 1.57 \), area beyond = 0.058.

Calculate Shaded Area

\[ \text{Shaded Area} = \text{Area Beyond } z_{60} - \text{Area Beyond } z_{69} \]

\[ \text{Shaded Area} = 0.386 - 0.058 = 0.328 \]

Conclusion

- 32.8% of students received a 2:1.
- We can work out how many people this was by multiplying the proportion by the number of scores collected:

\[ 130 \times 0.328 = 43 \text{ people} \]

Example 4

- The time taken for a lecturer to bore their audience to sleep was measured.
- The average time was 7 minutes, with a standard deviation of 2.
- What is the minimum time that the audience stayed awake for the most interesting 10% of lecturers?
Convert the Proportion to a Z-score

• 10% as a proportion is 0.10.
• Look in column labelled area beyond z.
• Find the value 0.10 in that column.
• Read off the corresponding z-score:

\[ z = 1.28 \]

Conclusion

• The time that cuts off the most entertaining 10% of lecturers is 9.5 minutes.